

Fast quantum algorithm for EC3 problem with trapped ions

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Adiabatic quantum computing (AQC) is based on the adiabatic principle, where a quantum system remains in an instantaneous eigenstate of the driving Hamiltonian. The final state of the Hamiltonian encodes solution to the problem of interest. While AQC has distinct advantages, recent researches have shown that quantumness such as quantum coherence in adiabatic processes may be lost entirely due to the system-bath interaction when the evolution time is long, and consequently the expected quantum speedup dose not show up. Here we propose a fast-signal assisted adiabatic quantum algorithm. We find that by applying a sequence of fast random or regular signals during the evolution process, the runtime can be reduced greatly, yet advantages of the adiabatic algorithm remain intact. Significantly, we present a *randomized* Trotter formula and show that the driving Hamiltonian and the sequence of fast signals can be implemented simultaneously. We apply the algorithm for solving the 3-bit exact cover problem (EC3) and put forward an approach for implementing the problem with trapped ions.

Introduction.— The adiabatic principle addresses quantum evolution governed by a slowly-varying Hamiltonian where the system will stay near its instantaneous ground state [1, 2]. It has a variety of applications in quantum information processing, such as adiabatic quantum computing [2], fault-tolerance against quantum errors [4], and universal holonomic quantum computation [5–7] based on the Berry's phase [8–10]. AQC is one of quantum computing models that have potential in solving certain problems much faster than their classical counterparts, in particular factoring large integers [11], searching unsorted database [12] and simulating quantum many-body problems [13].

Adiabatic quantum computation processes in a way where the quantum system time-evolves from the ground state of an initial Hamiltonian to that of the final Hamiltonian, which encodes the solution to the problem of interest. In AQC, a long evolution time guarantees that the final state reaches the ground state of the final problem Hamiltonian. This requires long coherence time in experimental implementation of the process. As such, the evolution time is crucial for AQC to be valid. If the evolution time is too long, quantumness may become vanishingly small and consequently the quantum speedup over classical computation will vanish. Recently an interesting experiment [3] has been performed to address the crucial question: whether or not a large-scale quantum device has the potential to outperform its classical counterpart. The experimental test was done for finding the ground state of an Ising spin glass model on the 503-qubit D-Wave Two system which are designed to be a physical realization of quantum annealing using superconducting flux qubits. There was no evidence found for quantum speedup. One of main reasons could be that the runtime is so long that before the end of an adiabatic quantum algorithm, decoherence has completely ruined

its quantumness.

Because of decoherence, a quantum algorithm with short runtime is always desired to keep its quantumness in the computational process. This is particularly important for adiabatic quantum algorithms since a strict adiabatic process requires an infinite runtime. In this paper, we present an approach that speeds up AQC substantially by applying fast signals in the dynamical evolution process, while keeping the high fidelity between the final state and the eigenstate of the problem Hamiltonian. This approach is experimentally feasible for implementing adiabatic quantum algorithms. We demonstrate this approach by solving a 3-bit exact cover problem (EC3).

The Algorithm.— The EC3 problem is a particular instance of satisfiability problem and is one of the NP-complete problems. No efficient classical algorithm has been found for solving this problem. On a quantum computer the EC3 problem can be formulated as follows [2, 15]: for a Boolean formula with M clauses

$$C_1 \wedge C_2 \wedge \cdots \wedge C_M, \quad (1)$$

where each clause C_l is true or false depending on the values of a subset of the n bits, and each clause contains three bits. The clause is true if and only if one of the three bits is 1 and the other two are 0. The task is to determine whether one (or more) of the 2^n assignments satisfies all of the clauses, and find the assignment(s) if it exists.

In Ref. [2, 15], a quantum adiabatic algorithm for solving the EC3 problem has been proposed. In this algorithm, the time-dependent evolution Hamiltonian $H_0(t)$ is

$$H_0(t) = J_0 [(1 - t/T) H_B + (t/T) H_P], \quad (2)$$

where H_B is the initial Hamiltonian whose ground state is used as the initial state, H_P is the Hamiltonian of

the EC3 problem whose ground state is the solution to the EC3 problem and T is the total evolution time or the runtime. Here J_0 is the strength of the Hamiltonian and is set as $J_0 = 1$ in this paper. In this algorithm, the Hamiltonian of the system evolves adiabatically from H_B to the problem Hamiltonian H_P , meaning that the system evolves from the ground state of H_B to the ground state of H_P . H_B is defined as

$$H_B = \sum_C H_{B,C}. \quad (3)$$

where $H_{B,C}$ is the Hamiltonian of clause C . Let i_C , j_C and k_C be the 3 bits associated with clause C . $H_{B,C}$ is defined as

$$H_{B,C} = H_B^{i_C} + H_B^{j_C} + H_B^{k_C}, \quad (4)$$

with

$$H_B^i = \frac{1}{2} (1 - \sigma_x^i) \quad (5)$$

and σ_x^i are the Pauli matrices. The Hamiltonian H_P for the EC3 problems is defined as follows: for each clause C , one can define an “energy” function

$$h_C(z_{i_C}, z_{j_C}, z_{k_C}) = \begin{cases} 0, & \text{if } (z_{i_C}, z_{j_C}, z_{k_C}) \text{ satisfies clause } C \\ 1, & \text{if } (z_{i_C}, z_{j_C}, z_{k_C}) \text{ violates clause } C \end{cases} \quad (6)$$

such that

$$H_P |z_1 z_2 \cdots z_n\rangle = h_C(z_{i_C}, z_{j_C}, z_{k_C}) |z_1 z_2 \cdots z_n\rangle, \quad (7)$$

where $|z_j\rangle$ is the j -th bit and has a value 0 or 1. Define

$$H_P = \sum_C H_{P,C}, \quad (8)$$

and we then have $H_P |\psi\rangle = 0$, if and only if $|\psi\rangle$ is in a superposition of states $|z_1 z_2 \cdots z_n\rangle$, where the bit string $z_1 z_2 \cdots z_n$ satisfies all of the clauses.

In what follows we will describe our approach for solving the EC3 problem by applying a sequence of fast pulses during the dynamical process [4]. We consider a Hamiltonian – a *dressed* $H_0(t)$,

$$H(t) = (1 + c(t)/J_0) H_0(t), \quad (9)$$

where $c(t)$ represents a sequence of fast signals. Ref. [4] shows that the adiabaticity can be enhanced and even induced by $c(t)/J_0$ – regular, random, and even noisy fast signals. Specifically, $c(t)/J_0$ could be a white noise signal in magnetic field, as exemplified in Ref. [4]. We will use this strategy to speed up adiabatic quantum algorithms and then illustrate our general approach by an experimentally feasible example.

We now come to explain the principle and experimental implementation of our approach in terms of a simple

but nontrivial EC3 problem. Consider a 4-bit EC3 problem, where we select the 3-bit set of clauses as $\{1, 2, 3\}$, $\{2, 3, 4\}$, and $\{1, 2, 4\}$. The solution to this problem is $|0100\rangle$.

For this specific model, we show numerically that when $T_0 > 160$, the system enters the adiabatic regime and $F \approx 0.999$ at $t = T$. In order to study the contributions of fast signals, we set $T = 40 < T_0$ in the non-adiabatic regime, and apply a sequence of fast regular pulses during the adiabatic process. The interval between pulses is set as 0.08, and the pulses strengths are changed $s = 0, 0.5, 1.0, 2.0$, respectively. Fig. 1 shows the dynamics of fidelity $F = |\langle \psi(t/T) | \psi_0(t/T) \rangle|$ between the system wave function and the instantaneous ground state of $H(t)$, where $\psi(t/T)$ is the wave function governed by the Schrödinger equation or the time-ordering evolution operator and $|\psi_0(t/T)\rangle$ represents the instantaneous ground state of the Hamiltonian $H(t)$. It is clear in the figure that as the strength of pulses increases, the adiabaticity is induced from a non-adiabatic regime and the fidelity F is approaching to one, in particular in the region where the solution is encoded. The quality of pulse control can also be improved by increasing the density of fast signals.

Different types of fast signals work as perfect as regular rectangular pulses [18]. Fig. 1b shows the fidelity dynamics by applying different fast signals, even random signals as in Fig. 3. The red dashed line shows the result by an even simpler fast signal $c(t) = 2 \cos^2(10t)$ and the blue dotted line is that of $2 \sin^2(10t)$. The black solid line uses regular rectangular pulses with $s = 2.0$ and $\Delta = 0.08$, as a reference.

Fast signals reduce the runtime of adiabatic evolution algorithms greatly, and keep very high fidelity F particularly when the system reaches our target – the ground state of the problem Hamiltonian H_P . Furthermore, the runtime can be even shorten for example to half, $T = 20$, with the pulses interval 0.04. We set the strength of pulses as $s = 0, 1.0, 2.0$, respectively, as in Fig. 2. It shows again that the adiabaticity is greatly enhanced by increasing strengths.

Adiabaticity can be induced from an originally fast dynamical process if pulses signals are even stronger. For example, if the signal strength $s = 15$, the system wave function evolves along the adiabatic path in the runtime $T = 9$ and at the very high fidelity $F = 0.999$ overlapping with the eigenstate of H_B , which is 17 times faster than the natural adiabatic process where the runtime $T_0 = 160$. Numerical analysis shows that if we are allowed to increase the strength at will, the runtime T can be as fast as we wish. Other examples are, if $s = 1.0, 5, 30$, $T \approx 70, 23, 5.0$, respectively.

Randomized Trotter formula and implementation of the algorithm in trapped ions. – We now discuss the feasibility to experimentally implement our algorithm on an ion trap quantum information processor.

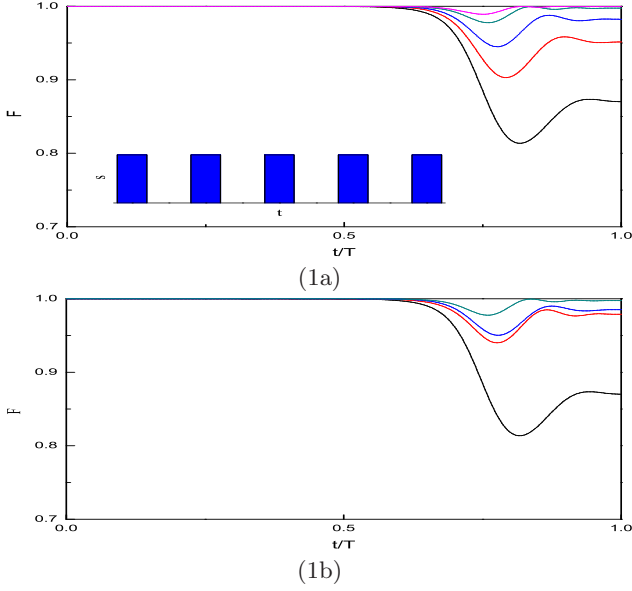


FIG. 1. (Color online) Dynamics of fidelity $F = |\langle \psi(t/T) | \psi_0(t/T) \rangle|$. The evolution time or runtime $T = 40$, the intervals between pulses are set as $\Delta = 0.08$. The black solid line shows the result for the strength $s = 0$; the red dashed line for $s = 0.5$; the blue dotted line for $s = 1.0$ and the dash-dot dark cyan line shows for $s = 2.0$. The magenta short dashed line shows the result of the evolution time $T_0 = 160$ when the system enters the adiabatic regime justified by $F(T) \geq 0.999$. For this model we consider this curve as a reference: *paths* $\psi(t/T)$ are in the adiabatic regime if $F(T) \geq 0.999$. (b). T and Δ are the same as in (a). The black solid line shows the dynamics of F for the case where no signal is applied; the red dashed line (the blue dotted line) shows the fidelity dynamics when applying fast signals $2\cos^2(10t)$ ($2\sin^2(10t)$), and the dark cyan dash-dot line shows $F(t)$ controlled by fast pulse signals with the pulse strength $s = 2.0$.

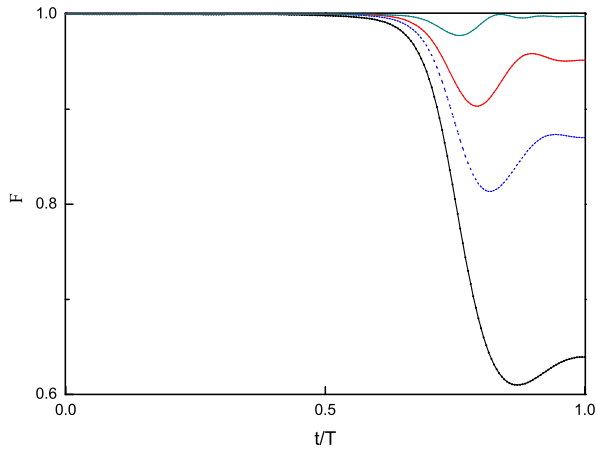


FIG. 2. (Color online) Dynamics of fidelity F with the runtime $T = 20$ and the intervals $\Delta = 0.04$. The black solid line shows the dynamics for $s = 0$; the blue dotted line for $s = 1.0$; the red dashed line shows the result for $s = 2.0$ and the dark cyan dash-dot line for $s = 5.0$.

In general, the EC3 problem Hamiltonian is supposedly stored in a Oracle and is called when it is needed. In order to perform experimental demonstration of our algorithm, here we simulate the 4-bit EC3 Hamiltonian with trapped ions. We first write the problem Hamiltonian explicitly in the qubit space,

$$H_P = \frac{3}{8} (\sigma_z^1 \sigma_z^2 \sigma_z^3 + \sigma_z^1 \sigma_z^2 \sigma_z^4 + \sigma_z^2 \sigma_z^3 \sigma_z^4) + \frac{1}{4} (\sigma_z^1 \sigma_z^2 + \sigma_z^2 \sigma_z^3 + \sigma_z^2 \sigma_z^4) + \frac{1}{8} (\sigma_z^1 \sigma_z^3 + \sigma_z^1 \sigma_z^4 + \sigma_z^3 \sigma_z^4) - \frac{1}{4} (\sigma_z^1 + \sigma_z^3 + \sigma_z^4) + \frac{15}{8}. \quad (10)$$

Note that the Hamiltonian contains up to three-body interactions, since symmetry rules out more complicated interactions which may appear in multi-bit EC3 problems.

The time-ordering evolution operators driven by the time dependent Hamiltonians $H(t)$ and $H_0(t)$ cannot be analogously simulated by trapped ions. Therefore digital simulation has to be employed. The standard recipe of digital simulation for adiabatic processes is the use of the Trotter formula, as done in previous literatures Ref. [17]. In what follows, we will present a *randomized* Trotter formula (RTF) to mimic $H(t)$, which effectively combine the two processes, applying fast signals during the dynamics and simulating $H_0(t)$.

The time-ordering unitary evolution operator is implemented as

$$U(k\tau) \approx e^{-iH_0(k\tau)\tau_k} \dots e^{-iH_0(2\tau)\tau_2} e^{-iH_0(\tau)\tau_1} \quad (11)$$

up to order $O(\tau^2)$ and where $j = 1, \dots, k$. Usually, the evolution operator of $H_0(k\tau)$ is simulated by setting all $\tau_j = \tau$.

The distinctive recipe of our RTF is that we set

$$\tau_j = \left(1 + \frac{c(j\tau)}{J_0}\right) \tau, \quad (12)$$

such that $e^{-iH_0(j\tau)\tau_j} = e^{-iH(j\tau)\tau}$. The equality links two different physical operations. The left is the simulated H_0 evolving during a short but uneven time interval τ_j , and the right means a fast signal $c(j\tau)$ has been implemented, at the time instance $j\tau$, upon H_0 that *transforms* into the dressed H evolving in an even time interval τ . The mathematical equivalence implies that we can experimentally simulate $e^{-iH_0(j\tau)\tau_j}$ instead of $e^{-iH(j\tau)\tau}$, whose simulation ingredient is not yet known (*unknown* for this model but it is simple to implement $c(t)$ upon H_0 for most of systems, such as an additional magnetic fast-varying field upon spins). In other words, the simulation (11) for H_0 becomes that of H ,

$$U(k\tau) \approx e^{-iH(k\tau)\tau} \dots e^{-iH(2\tau)\tau} e^{-iH(\tau)\tau} \quad (13)$$

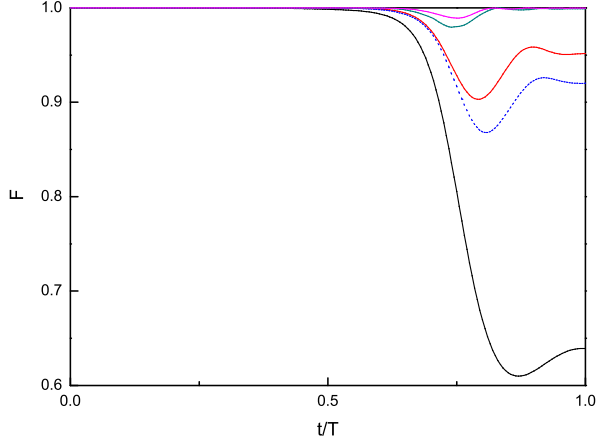


FIG. 3. (Color online) Dynamics of $F(t)$ with the evolution time $T = 20$. The black solid line shows F without signal control; the red dashed line is F controlled by a sequence of fast pulses with $s = 2.0$ and $\Delta = 0.04$; the blue dotted and the dark cyan dash-dot lines show the dynamics of F obtained by the simulated Eq. (11), where τ_j varies randomly in the range $[2.0\tau, 3.0\tau]$ and $[4.0\tau, 8.0\tau]$, respectively. The magenta short dashed line is the $T_0 = 160$ reference for the adiabatic regime.

up to order $O(\tau^2)$.

The evolution operator of H_0 is simulated by the Trotter decomposition,

$$e^{-iH_0(j\tau)\tau_j} \approx e^{-iH_B(1-j/k)\tau_j} e^{-iH_P(j/k)\tau_j}. \quad (14)$$

Experimentally, exact control of these uneven time intervals τ_j might not be easy. Therefore, the easiest way for experimentalists is to assign random values to these intervals τ_j . This is equivalent to employ random fast signals $c(j\tau)$, which has shown the same excellent control quality as that of other fast signals [4, 18].

We set the runtime $T = 20$ and let τ_j change randomly in the range $[2.0\tau, 3.0\tau]$ and $[4.0\tau, 8.0\tau]$ respectively, and perform simulation. Fig. 3 shows the results and compares them with regular pulses. It is clear that random fast signals work as perfect as regular pulses. When the variation range of τ_j is larger, the enhancement to adiabaticity is even better than that of fixed τ_j 's, and evolves on the same adiabatic path as that of the adiabatic reference where $T_0 = 160$.

Now we come to discuss the experimental implementation on trapped ions. It is clear that we need only to implement the slices $e^{-iH_B(1-j/k)\tau_j} e^{-iH_P(j/k)\tau_j}$ and repeat them to perform the evolution operator $U(k\tau)$ until $k\tau = T$. H_B is a simple single-qubit Hamiltonian and can be implemented on most of sophisticated quantum devices, including trapped ions. It is a challenge for quantum devices to implement three or more body interactions. Fortunately, trapped ions do not have this difficulty. Consider tensor products of Pauli matrices in the form of $A = \prod_{m=1}^n \sigma_\alpha^m$ with $\sigma_\alpha^m \in \{1, \sigma_x^m, \sigma_z^m\}$.

These products can be implemented efficiently with the Mølmer-Sørensen (MS) scheme [19] on trapped ions,

$$e^{i\phi\sigma_x^1 \otimes \sigma_x^2 \otimes \dots \otimes \sigma_x^n} = U_{MS}(-\pi/2, 0) U_{anc}(\phi) U_{MS}(\pi/2, 0), \quad (15)$$

where the exponential is implemented by two MS gates to the n system ions and an ancilla qubit (no. 0), $U_{MS}(\theta, \varphi) = \exp\left(-i\frac{\theta}{4}(\cos\varphi S_x + \sin\varphi S_y)^2\right)$, and $S_{x,y} = \sum_{j=0}^n \sigma_{x,y}^j$. $U_{anc}(\phi)$ is defined as when n is odd, $U_{anc}(\phi) = e^{-i\phi\sigma_0^y}$ for $n = 4m + 1$, and $U_{anc}(\phi) = e^{i\phi\sigma_0^y}$ for $n = 4m - 1$, and when n is even, $U_{anc}(\phi) = e^{i\phi\sigma_0^z}$ for $n = 4m$, $U_{anc}(\phi) = e^{-i\phi\sigma_0^z}$ for $n = 4m - 2$.

Discussion.— A short runtime is of crucial importance for adiabatic quantum algorithms to achieve polynomial time speedups over their classical counterpart, because it is difficult to keep quantumness of a system for long time in presence of noisy environment. In this paper, we propose an adiabatic quantum algorithm assisted with fast signal and show that by applying a sequence of fast signals, the runtime in the adiabatic quantum computing can be greatly reduced. This technique has practical interest in the physical implementation of adiabatic quantum algorithms. We applied this approach to solving the EC3 problem and discuss the feasibility to implement it on trapped ions. We introduce a randomized Trotter formula which effectively implements effects of fast signals upon the original Hamiltonian, which, as we show, can be implemented efficiently on a trapped ion system.

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Strength for Speed: Expedited Adiabatic Process

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Adiabatic theorem ensures that a quantum system remains on its adiabatic path: an instantaneous eigenstate of the driving Hamiltonian, and provides the theoretical basis of adiabatic quantum information processor (AQIP). A functional large scale AQIP is one of the most promising candidates for the ultimate universal quantum computer [1], where each algorithm is designed to run through a specifically programmed adiabatic path. While adiabatic processor is claimed to have great advantages [2], recent experiments on the D-Wave Two system found no evidence of quantum speedup [3]. One of main reasons for the dysfunction could be that the runtime is so long that before the end of adiabatic quantum path, decoherence has completely ruined the quantumness. Speedup of programmed adiabatic paths is crucial in realization of practical large scale quantum computation. Here we find that a time scaling transformation can transfer strength of the driving Hamiltonian into speed of the adiabatic process. We prove rigorously that if it has strong enough strength, a generic non-adiabatic Hamiltonian can become an adiabatic Hamiltonian in the scaling time domain. This offers in principle unlimited speedup for passing through adiabatic paths.

Quantum adiabatic theorem can be formulated as follows: in the adiabatic regime, the Schrödinger equation

$$H_0\left(\frac{t}{T_0}\right)\psi(t) = i\partial_t\psi(t) \quad (16)$$

has the solution $\psi(t) = e^{i\theta_n(t)}|E_n(t)\rangle$ if initially $\psi(0) = |E_n(0)\rangle$, where $|E_n(t)\rangle$'s are instantaneous eigenstates of $H_0(\frac{t}{T_0})$. Here $e^{i\theta_n(t)}$ is a phase factor and T_0 is the total evolution time or runtime characterizing the duration needed for $H_0(\frac{t}{T_0})$ to become adiabatic, which is determined by the standard adiabatic conditions.

Consider a Hamiltonian $H(\frac{t}{T}) = JH_0(\frac{t}{T})$, where $J > 1$ and $T < T_0$ such that $H_0(\frac{t}{T})$ is not adiabatic. The Hamiltonian H has stronger strength than H_0 . Physically, for instance the strong strength of the Hamiltonian of a spin in a magnetic field can be made by increasing the amplitude of the magnetic field. The Schrödinger equation for H is $JH_0(\frac{t}{T})\psi'(t) = i\partial_t\psi'(t)$.

We now apply a time scaling transformation $\tau = Jt$ such that the Schrödinger equation of H becomes

$$H_0\left(\frac{\tau}{JT}\right)\psi'(\tau/J) = i\partial_\tau\psi'(\tau/J), \quad (17)$$

in the scaling time τ domain, where the Hamiltonian and time derivative ∂_τ have the exact same profile as those in the Schrödinger equation (16) when $JT = T_0$. Therefore, the Schrödinger equation of H in the scaling time domain is identical to that of H_0 in the real time t domain, such that their solutions are identical,

$$\psi'(\tau/J) \equiv \psi(\tau) = e^{i\theta_n(\tau)}|E_n(\tau)\rangle. \quad (18)$$

This states a *quantum adiabatic theorem in the scaling time domain*, and is also a formal proof that the runtime of an adiabatic quantum process can be *reduced J times* or decreases from T_0 to $T = T_0/J$. Note that the standard adiabatic conditions remain unchanged in the scaling time domain, except t being replaced by τ .

In analogy with the normal adiabatic theorem for adiabatic processes, the quantum adiabatic theorem in the scaling time domain provides the basis of expedited adiabatic processes.

An example is the adiabatic algorithm in Ref. [2], where

$$H(t/T) = J[(1 - t/T)H_B + (t/T)H_P]. \quad (19)$$

The expedited adiabatic process runs on the adiabatic path in the scaling time domain: $|E_0(t/J)\rangle$ (note that τ is replaced by t/J), where the initial state is ground state of H_B and the final state is ground state of H_P encoding solution to the problem of interest. The evolution from H_B to H_P needs only $T = T_0/J$, for example, as a limit the runtime of an adiabatic algorithm could be $T \approx 0$ if $J \rightarrow \infty$.

The quantum adiabatic theorem in the scaling time domain clearly suggests the strategy of experimentally implementing an expedited adiabatic processes: *simply pushing the strength J to its upper bound* (and combining with fast signal proposal to further improve the adiabaticity as in Ref. [4, 5]). The quantum adiabatic theorem offers in principle unlimited speedup (non relativistic) for passing through adiabatic paths when the strength J is very strong.

Acknowledgments

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